Computer models of motion: Iterative calculations

OBJECTIVES

In this activity you will learn how to:

- Create 3D box objects
- Update the position of an object iteratively (repeatedly) to animate its motion
- Update the momentum and position of an object iteratively (repeatedly) to predict its motion

TIME

You should plan to finish this activity in 50 minutes or less.

GROUP ROLES

Before you begin, you should agree on the responsibilities of the Manager, the Recorder, and the Skeptic for this activity.

COMPUTER PROGRAM ORGANIZATION

A computer program consists of a sequence of instructions.
The computer carries out the instructions one by one, in the order in which they appear, and stops when it reaches the end.
Each instruction must be entered exactly correctly (as if it were an instruction to your calculator).
If the computer encounters an error in an instruction (such as a typing error), it will stop running and print a red error message.

A typical program has four sections:

1. Setup statements
2. Definitions of constants (if needed)
3. Creation of objects and specification of initial conditions
4. Calculations to predict motion or move objects (done repetitively in a “loop”)

I. Setup statements

Using IDLE for VPython, create a new file and save it to your own K drive. Make sure to add ".py" to the file name.

Enter the following two statements in the IDLE editor window:

```
from __future__ import division
from visual import *
```

Remember that every VPython program begins with these setup statements.
The first statement (from *space* underscore underscore future underscore underscore *space* import *space* *) tells the Python language to treat 1/2 as 0.5. Without the first statement, the Python programming language does integer division with truncation and 1/2 is zero!
The second statement tells the program to use the 3D module (called “visual”).

2. Constants

Following the setup section of the program you would define physics constants. We'll talk about this in later projects.

3a. Creating objects

Create a box object to represent the track:

```
track = box(pos=vector(0,-.05, 0), size=(2.0, 0.05, .10), color=color.white)
```

Run the program by pressing F5. Arrange your windows so the Python Shell window is always visible.
Kill the program by closing the graphic display window.

Now create a second box object, named "cart", with some color other than white. Give this object a position (pos) of **`(0,0,0)`** and a size of **`(0.1, 0.04, 0.06)`**.

Run the program by pressing F5. Zoom (both mouse buttons down) and rotate (right mouse button down) to examine the scene. The cart should be floating just above the track. Is it?

Reposition the cart so its left end is aligned with the left end of the track.
To do this you will have to answer the following questions:
Where is the "pos" of a box object? The left end? The right end? The center?
Do the numbers in the "size" of a box refer to the total length, or the distance from the center to one edge?
You can answer these by experimentation, or by looking in the online reference manual (Help menu, choose Visual, click on Reference Manual.)

3b. Initial conditions

Any object that moves needs two vector quantities declared before the loop begins:

1. initial position; and
2. initial momentum.

You've already given the cart an initial position at the left end of the track. Now you need to give it an initial momentum. If you push the cart with your hand, the initial momentum is the momentum of the cart just *after* it leaves your hand. Since the definition of momentum at speeds much less than the speed of light is $\vec{p} \approx m\vec{v}$, we need to tell the computer the cart's mass and the cart's initial velocity.

- **Below the existing lines of code, type the following new lines:**

```
mcart = 0.80
pcart = mcart*vector(0.5, 0, 0)
print 'cart momentum =', pcart
```

We have made up a new variable named "**`mcart`**" The symbol **`mcart`** now stands for the value 0.80 (a scalar), which represents the mass of the cart in kilograms.
We have also created a new vector variable **`pcart`**, which is the momentum of the cart. We assigned it the initial value of $(0.80\ \text{kg})\langle 0.5, 0, 0\rangle\ \text{m/s}$.

- **Run the program. Look at the Python Shell window. Is the correct value of the vector pcart printed there? From what is printed, how can you tell it is a vector?**

Note: There are no "built in" physics attributes **`p`** or **`m`** for objects like there are built-in geometrical attributes **`pos`** or **`radius`**. However, Python allows us to create new attributes for objects. We could have called the momentum **`cart.p`**, or the mass **`cart.m`**, instead of **`pcart`** or **`mcart`**. It can be helpful to create attributes like mass or momentum associated with objects so we can easily tell apart the masses and momenta of different objects in a complex program.

3b. Time step and total elapsed time

To make the cart move we will use the position update equation $\vec{r}_f = \vec{r}_i + \vec{v}\Delta t$ repeatedly in a "loop". We need to define a variable **`deltat`** to stand for the time step Δt, and a variable **`t`** to stand for the total time elapsed since the motion started. Here we will use the value $\Delta t = 0.01$ s.

- **Type the following new lines at the end of your program:**

```
deltat = 0.01
t = 0
```

This completes the first part of the program, which tells the computer to:

a. Create numerical values for constants we might need (none were needed this time)
b. Create 3D objects
c. Give them initial positions and momenta

4. Repeated calculations: Loops

In a computer program a sequence of instructions that are to be repeated is called a loop. The kind of loop we will use in VPython starts with a "while" statement. Instructions inside the loop are indented. IDLE will indent automatically after you type a colon.

- **To write a simple loop, type the following new lines at the end of your program.**
- **Be sure to type a colon (:) at the end of the while statement.**
- **Make sure the indenting is correct, as shown below, then run:**

```
while t < 0.2:
  print 'the time is now', t
  t = t + deltat
print 'after the loop'
```

The statement:

```
t = t + deltat
```

may look like a mathematical error. However, in a program, the "=" sign has a different meaning than in a mathematical equation.
The right hand side of the statement tells Python to read up the old value of **`t`**, and add the value of **`deltat`** to it. The left hand side of the statement tells Python to store this new value into the variable **`t`**.

- **Run the program. Look at the Python Shell window.**

Look at the printed output in the Shell window. Answer the following questions:
What makes the loop stop? Why is the first printed time 0? Why is the last time 0.19 and not 0.2?
How can you get the program to print values from 0 through 0.3? (Try it.)

4a. Constant momentum motion

Consider a cart moving with constant momentum. Somebody or something gave the cart some initial momentum. We're not concerned here with how it got that initial momentum. We'll predict how the cart will move in the future, *after* it acquired its initial momentum.

You will use your iterative calculational "loop". Each time the program runs through this loop, it will do two things:

1. Use the cart's current momentum to calculate the cart's new position
2. Increment the cumulative time **`t`** by **`deltat`**

You know that the new position of an object after a time interval Δt is given by

$$\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}}\Delta t$$

where $\vec{r}_f$ is the final position of the object, and $\vec{r}_i$ is its initial position. If the time interval Δt .is very short, so the velocity doesn't change very much, we can use the initial or final velocity to approximate the average velocity.

Since at low speed $\vec{p} \approx m\vec{v}$, or $\vec{v} \approx \vec{p}/m$, we can write

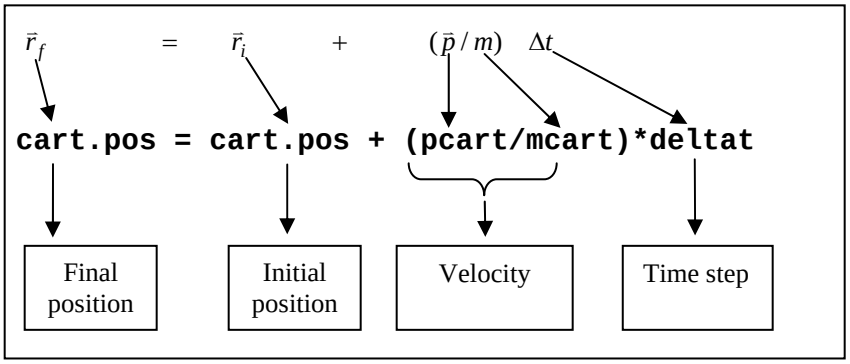
$$\vec{r}_f = \vec{r}_i + (\vec{p}/m)\Delta t$$

We will use this equation to increment the position of the cart in the program. First, we must translate it so VPython can understand it.

- **Delete or comment out the line inside your loop that prints the value of t .**
- **On the indented line after the "while" statement, and before the statement updating t, type the following:**

```
cart.pos = cart.pos + (pcart/mcart)*deltat
```

Notice how this statement corresponds to the algebraic equation:



Think about the situation and answer the following question:
What will the elapsed time **`t`** be after moving two meters?

- **Change the while statement so the program runs just long enough for the cart to travel 2 meters.**
- **Now, run the program. What do you see?**

Slowing down the animation

When you run the program, you should see the cart at its final point. The program is executed so rapidly that the entire motion occurs faster than we can see, because a "virtual time" in the program elapses much faster than real time does. We can slow down the animation rate by adding a "rate" statement.

- **Add the following line inside your loop (indented):**

```
rate(100)
```

Every time the computer executes the loop, when it reads "**`rate(100)`**", it pauses long enough to ensure the loop will take 1/100th of a second. Therefore, the computer will only execute the loop 100 times per second.

- **Now, run the program.**

You should see the cart travel to the right at a constant velocity, ending up 2 meters from its starting location.

Note: The cart going beyond the edge of the track isn't a good simulation of what really happens, but it's what we told the computer to do. There are no "built-in" physical behaviors, like gravitational force, in VPython. Right now, all we've done is tell the program to make the cart move in a straight line. If we wanted the cart to fall off the edge, we would have to enter statements into the program to tell the computer how to do this.

Answer the following questions:

1. Which statement in your program represents the position update formula?
2. What would you have to change in your program to make the cart start at the right end of the track and move to the left? Do this. When you have succeeded, compare your program to that of another group.

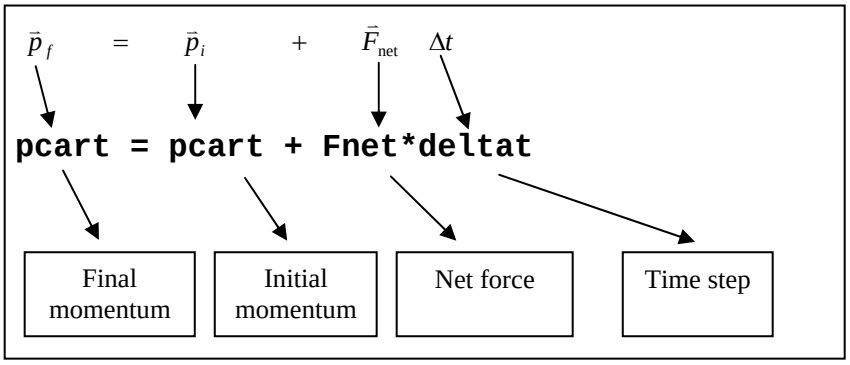
4b. Changing momentum

Your running program should now have a model of a cart moving at constant velocity from right to left along a track.

- **What should happen to the motion of the cart if you apply a constant force to the right? Discuss this among your group, and write down your prediction.**

As discussed in Chapter 3 of the textbook, an iterative prediction of motion (summarized on page 87) can include the effects of forces that change the momentum of an object.

Here is how the Momentum Principle can be translated into VPython:



- **Before the loop, create a new vector variable named `Fair`, and assign it the value < 0.3,0,0 >, using the appropriate VPython syntax (look at how you created a variable to represent the momentum vector).**
- **Increase the time the loop runs, allowing the cart to move for 20 seconds, in order to explore the following questions:**

Answer the following questions:

1. Should the Momentum Principle statement be placed before the loop or inside the loop?
2. How should you write this statement in order to use the force Fair (the constant force by the air on the fancart) as the net force in this statement? Do this. Does your program work as you predicted it should? If not, fix it (you may wish to discuss your approach with another group.)
3. By experimentation, determine a value for Fair, the force by the air on the fancart, that produces the following motion:

The cart starts at the right end of the track and moves to the left, gradually slowing down, and coming to a stop near the left end of the track; it turns around and moves to the right, speeding up. (Note that you will need to let the loop run longer in order to see this behavior.)

4c. 2D motion

In a computer program you can model behavior that would be difficult to observe in the real world.

Do the following:

Change the initial momentum of the fancart so that it includes a +y component similar in magnitude to the x component of the momentum. What happens? Explain this, then compare your explanation to that of a neighboring group.

5. Turn in your program to WebAssign

Make sure everyone in the group agrees that the program is correct. Check with a neighboring group.

The Recorder should turn in the program to WebAssign, and make sure everyone has a copy of the program.

When you turn in a program to WebAssign, be sure to follow the instructions given there, which may sometimes ask you to change some of the parameters in your program. The Recorder should email copies to everyone before leaving the lab.

Using VPython on your own

VPython is free. You can download VPython from http://vpython.org and install it on your own computer. VPython is also available in the campus public clusters in the Math, Statistics, and Physics section of the Novell Application Launcher; doubleclick "IDLE for VPython".

In the text editor (IDLE), on the Help menu you can choose “Visual”, then “Reference manual”, or choose “Python Docs” to obtain detailed information on the Python programming language upon which VPython is based. We will use only a small subset of Python’s extensive capabilities.

6. Playing around (not required)

What would you need to do to include the effects of gravity? The gravitational force near the Earth is $\langle 0, -mg, 0 \rangle$, where g = +9.8 N/kg. Give the cart an initial velocity of $\langle -0.7, 3.0, 0 \rangle$ m/s. Turn on gravity and turn off the fan force, use a smaller deltat of 0.005 for accuracy, and use an “if” statement to make the cart bounce on the track:

```
if cart.pos.y < 0:
   cart.p.y = abs(cart.p.y) # make the cart move upward
```

Eventually you’ll see the cart bouncing off nothing! Can you think how to change your program to do something more realistic? Hint: You can combine logical tests by using the keywords “and” and “or”. For example, the logical expression “(y < 3) and (z > 2)” is true only if both of these conditions are true.